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Geometrical optics

Optics  $\Rightarrow$  The branch of Physics which deals with the study of phenomenon related to light.

Ex  $\Rightarrow$  Light, reflection, Dispersion, diffraction

Type of optics

- (i) Physical optics
- (ii) Geometrical optics

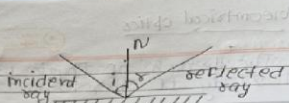
(i) Physical optics  $\Rightarrow$  It deals with the phenomenon based on the nature of light whether wave or particle or nature.

Ex  $\Rightarrow$  Interference, diffraction etc

(ii) Geometrical optics  $\Rightarrow$  It is related to study of optical instrument without taking into account of nature of light.

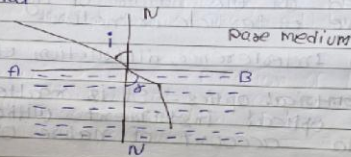
Ex  $\Rightarrow$  It becoming only reflection and refraction.

Reflection of light  $\Rightarrow$  Reflection of light is the phenomenon of bouncing back of light in the same medium on the surface of any object. In reflection path of light ray change without any change in medium.

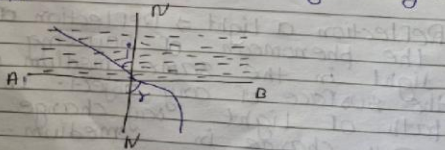


Reflection of light  $\Rightarrow$  When light passes from one to another medium then it deviate from its original path. So, this phenomenon is called reflection of light.

When light passes from <sup>rare</sup> medium to denser medium then it bends towards the normal.



When light passes from denser medium to rare medium then it goes away from the normal.



## # Fermat's Principle

In 1658, Pierre De Fermat a French mathematician stated that "A ray of light is passing from one point to another point through a set of media by any member of reflection or refraction choose a path along which the time taken is minimum or least."

It is also known as least time.



Based on this principle the law of rectilinear propagation, the law of reflection can be derived. However in some cases it has been found by time taken but light is not minimum but maximum or else. It is neither maximum nor minimum but it is stationary constant. Therefore modify form of Fermat's principle of least time is known as Fermat's principle of stationary time or Fermat's principle of extremum which may be stated as follows:

## \* Fermat's principle of stationary time or Fermat's principle of extremum path

A ray of light passing from one point to another point through a set of media by any member of reflection or refraction choose a path for which the time taken is either minimum or maximum or stationary time.

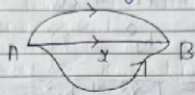
Notes: (i) Minimum: → In which case a ray of light plane reflecting or refracting plane through from one point to second point.

(ii) Maximum: → When refraction of light passing through a concave & convex lens image formation of lens time taken is maximum.

(iii) Stationary: → When reflection through elliptical reflector.

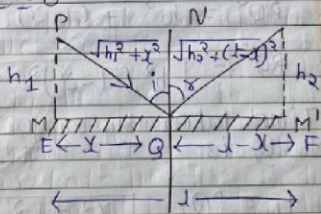
Derivation of law of reflection from Fermat's principle:

Fermat's Principle states that light travel b/w two points along the path that requires the least time as compared to the other nearby paths.



To minimize the time we set the derivative of the time with respect to  $x$  is equal to zero.

$$\text{i.e., } \frac{dt}{dx} = 0$$



Consider a plane reflecting surface  $MM'$ . A ray of light starting from point 'P' reflects off the surface  $MM'$  at point Q before arriving at point R.

$$\text{Let } PE = h_1, \quad PF = h_2$$

$$\text{Let } EF = l, \quad EQ = x$$

$$\text{then } QE = (l-x)$$

From right angled  $\triangle PEQ$  we have

$$PQ^2 = PE^2 + EQ^2$$

$$PQ^2 = h_1^2 + x^2$$

$$PQ = \sqrt{x^2 + h_1^2} \quad \text{--- (i)}$$

From right angle  $\triangle RFQ$  we have

$$RQ^2 = RF^2 + FQ^2$$

$$RQ^2 = h_2^2 + (l-x)^2$$

$$RQ = \sqrt{h_2^2 + (l-x)^2} \quad \text{--- (ii)}$$

Time taken by ray of light from P to Q is

$$t_1 = \frac{PQ}{c} = \frac{\sqrt{x^2 + h_1^2}}{c} \quad \text{--- (iii)}$$

Time taken by ray of light from Q to R is given by

$$t_2 = \frac{QR}{c}$$

$$t_2 = \frac{h_2^2 + (l-x)^2}{c} \quad - (iv)$$

Total time taken by ray of light in travelling from point P to B after reflection at point Q from reflecting surface MM' is given by

$$t = t_1 + t_2$$

$$t = \frac{\sqrt{h_1^2 + x^2}}{c} + \frac{\sqrt{h_2^2 + (l-x)^2}}{c}$$

To minimize the time use set the derivative of time w.r.t 'x' equal to zero.

$$\frac{dt}{dx} = \frac{1 \cdot 2x}{2c\sqrt{h_1^2 + x^2}} + \frac{-2(l-x)(c-1)}{2c\sqrt{h_2^2 + (l-x)^2}} = 0$$

$$\frac{dt}{dx} = \frac{x}{c\sqrt{h_1^2 + x^2}} - \frac{(l-x)}{c\sqrt{h_2^2 + (l-x)^2}} = 0$$

$$\frac{dt}{dx} = \frac{x}{\sqrt{h_1^2 + x^2}} - \frac{(l-x)}{\sqrt{h_2^2 + (l-x)^2}} = 0$$

$$\frac{x}{\sqrt{h_1^2 + x^2}} = \frac{(l-x)}{\sqrt{h_2^2 + (l-x)^2}} = 0$$

$$\frac{x}{\sqrt{h_1^2 + x^2}} = \frac{l-x}{\sqrt{h_2^2 + (l-x)^2}}$$

$$\sin i = \sin r$$

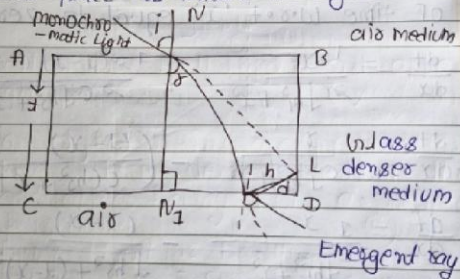
$$i = r \quad \text{Proved.}$$

$$\sin i = \frac{P}{H} = \frac{x}{\sqrt{h_1^2 + x^2}}$$

$$\sin r = \frac{P}{H} = \frac{l-x}{\sqrt{h_2^2 + (l-x)^2}}$$

\* Refraction of light through thin glass plate

When monochromatic light is incident on thin glass plate then refraction takes place as shown in figure



Lateral shift (d): The perpendicular distance b/w emergent ray and incident ray is called lateral shift.

# Expression for lateral shift

In  $\Delta PNL$

$$\sin(i-\delta) = \frac{PL}{PN}$$

$$\sin(i-\delta) = \frac{d}{PN}$$

$$\therefore d = PN \sin(i-\delta) \quad \text{--- (i)}$$

In  $\Delta PNM'$

$$\cos r = \frac{NM'}{PN}$$

$$\cos r = \frac{t}{PN}$$

$$\Rightarrow PN \cos r = t$$

$$\Rightarrow PN = \frac{t}{\cos r}$$

$$\therefore PN = \frac{t}{\cos r}$$

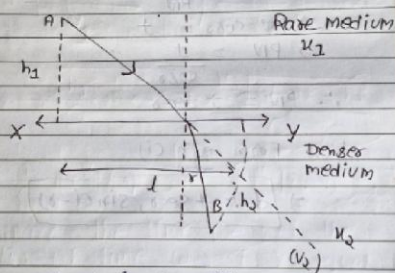
Now from eqn (i)

$$\Rightarrow d = \frac{t \sec r}{\cos r} \sin(i-\delta)$$

# Derivation of law of refraction from Fermat's principle :-

"Fermat's principle states that light travels b/w two points along the path that require the least time as compared to the other nearby paths"

To minimize the time we set the derivative of the time with respect to 'x' is equal to zero i.e.  $\frac{dt}{dx} = 0$



In Figure light is travelling from A to B. First in a medium of refractive index ( $n_1$ ) (speed =  $v_1$ ) & then in a medium of refractive index ( $n_2$ ) (speed =  $v_2$ ) through point p.

Time taken by ray of light is travelling to A' to P' is given by  $t_1 = \frac{AP}{v_1}$

$$\therefore t_1 = \frac{AP}{v_1}$$

$$t_1 = \frac{\sqrt{(h_1)^2 + (x)^2}}{v_1}$$

$$= \frac{\sqrt{(h_1)^2 + (x)^2}}{v_1} \quad \text{--- (i)}$$

Time taken by ray of light travelling from P to B is given

$$t_2 = \frac{PB}{v_2}$$

$$= \frac{\sqrt{(PF)^2 + (BF)^2}}{v_2}$$

$$= \frac{\sqrt{(L-x)^2 + (h_2)^2}}{v_2} \quad \text{--- (ii)}$$

Total time taken by ray of light is travelling from A to B after refraction at point P' is given by

$$t = t_1 + t_2$$

$$t = \frac{\sqrt{(h_1)^2 + (x)^2}}{v_1} + \frac{\sqrt{(L-x)^2 + (h_2)^2}}{v_2} \quad \text{--- (iii)}$$

$$v_2 = \frac{c}{\mu_2} \Rightarrow v_1 = \frac{c}{\mu_1} \quad \text{--- (iv)}$$

$$\left[ \text{Refractive index} = \frac{\text{Velocity of Light in air}}{\text{Velocity of Light in medium}} \right]$$

$$\text{Similarly, } \mu_2 = \frac{c}{v_2} \Rightarrow v_2 = \frac{c}{\mu_2} \quad \text{--- (v)}$$

Using eq (4) & (5) in eq (3)

$$t = \frac{\sqrt{(h_1)^2 + (x)^2}}{c/\mu_1} + \frac{\sqrt{(L-x)^2 + (h_2)^2}}{c/\mu_2}$$

$$t = \frac{\mu_1 \sqrt{h_1^2 + x^2}}{c} + \frac{\mu_2 \sqrt{(L-x)^2 + h_2^2}}{c}$$

$$t = \frac{1}{c} (\mu_1 \sqrt{h_1^2 + x^2} + \mu_2 \sqrt{(L-x)^2 + h_2^2})$$

To minimize the time we set the derivation of time w.r.t.  $x$  equal to

zero.

$$\frac{dt}{dx} = \frac{1}{c} \left( \mu_1 \frac{1}{2} \cdot \frac{2x}{\sqrt{h_1^2 + x^2}} + \mu_2 \cdot \frac{1}{2} \cdot \frac{2(L-x)(-1)}{\sqrt{(L-x)^2 + h_2^2}} \right)$$

= 0

$$= \frac{1}{c} \left[ \frac{v_1 x}{\sqrt{h_1^2 + x^2}} - \frac{v_2 (1-x)}{\sqrt{(1-x)^2 + h_2^2}} \right] = 0$$

$$= \frac{v_2 x}{\sqrt{h_1^2 + x^2}} - \frac{v_2 (1-x)}{\sqrt{(1-x)^2 + h_2^2}} = 0$$

$$= \frac{v_2 x}{\sqrt{h_1^2 + x^2}} = \frac{v_2 (1-x)}{\sqrt{(1-x)^2 + h_2^2}} = 0$$

$$= \frac{v_2 x}{\sqrt{h_1^2 + x^2}} = \frac{v_2 (1-x)}{\sqrt{(1-x)^2 + h_2^2}} = 0$$

$$\sin i = \frac{p}{h} = \frac{x}{\sqrt{(h_1)^2 + (x)^2}} \quad \sin r = \frac{p}{h} = \frac{1-x}{\sqrt{(1-x)^2 + (h_2)^2}}$$

$\Rightarrow \boxed{v_1 \sin i = v_2 \sin r}$  which is Snell's law of refraction.

Hence proved.

Refractive index  $\Rightarrow$  The ratio of velocity of light in vacuum to the velocity of light in medium is called refractive index of that medium.

$$\text{i.e., } \mu = \frac{c}{v}$$

Where,  $c$  = Speed of light ( $3 \times 10^8 \text{ m/s}$ )  
 $v$  = Velocity of light in a medium

Refractive index is unit less and dimensionless quantity. Its value is different in different medium.

e.g.,  
 air = 1  
 Water =  $\frac{4}{3}$   
 Glass =  $\frac{3}{2}$   
 Diamond = 2.42

Note: Refractive index of dense medium is greater than that of rare medium.

Relative refractive index ( $\mu_{21}$ ):-

When light passes from one medium to another medium the refractive index of medium (2) with respect to medium (1) is written as:

$$\mu_{21} = \frac{\mu_2}{\mu_1}$$

$$\text{Eg} \rightarrow \mu_{wg} = \frac{\mu_g}{\mu_w} = \frac{3/2}{4/3} = \frac{3}{2} \times \frac{3}{4} = \frac{9}{8}$$

$$\mu_{gw} = \frac{\mu_w}{\mu_g} = \frac{4/3}{3/2} = \frac{4}{3} \times \frac{2}{3} = \frac{8}{9}$$



critical angle  $\Rightarrow$  when light incident from dense medium of the refracting surface in such way that angle of refraction becomes  $90^\circ$  then corresponding angle of incident is called critical angle. It depend upon refractive index of the medium.

V.V.T \* Expression for critical angle

Let us consider a light incident from denser medium on the refracting surface and refraction takes place as shown in figure

According to Snell's law

$$n_1 \sin i = n_2 \sin r$$

$$\Rightarrow n_1 = \frac{\sin r}{\sin i}$$

where  $r = 90^\circ$ , then  $i = c$

$$n = \frac{\sin 90^\circ}{\sin c}$$

where  $r = 90^\circ$ , then  $i = c$

$$n = \frac{\sin 90^\circ}{\sin c}$$

$$n = \frac{1}{\sin c}$$

$$n \sin c = 1$$

$$\sin c = \frac{1}{n}$$

$$\therefore c = \sin^{-1} \frac{1}{n}$$

$\therefore$  Critical angle is inversely proportional to the refractive index.

### \* Total internal reflection

When light incident from denser medium on the refracting surface at greater angle of critical angle then whole light is reflected back in the same medium so this phenomenon is called Total internal reflection of light.

→ Condition for TIR: -

- (i) The light must travel from denser to rarer medium.
- (ii) Angle of incidence must be greater than critical angle.

→ Application of TIR of light

- i) Air bubble inside the water is more shining.
- ii) Diamond is more shining due to total internal reflection of light.

### \* Lateral shift (d)

The perpendicular distance between emergent ray and incident ray is called lateral shift.

Expression for lateral shift

In  $\triangle PNL$

$$\sin(i - r) = \frac{PL}{PN}$$

$$\sin(i - r) = \frac{d}{PN}$$

$$\therefore d = PN \sin(i - r) \quad \text{--- (i)}$$

In  $\triangle PNN'$

$$\cos r = \frac{NN'}{PN}$$

$$\cos r = \frac{t}{PN}$$

$$\Rightarrow PN \cos r = t$$

$$\Rightarrow PN = \frac{t}{\cos r}$$

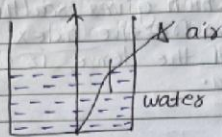
$$\therefore PN = t \sec r$$

Now from eqn (i)

$$\Rightarrow d = t \sec r \sin(i - r)$$

Relation b/w real depth and apparent depth:

When object is in denser medium and looking from air medium (rare) then it appear lifted from its original position.



$$\mu = \frac{\text{Real depth}}{\text{Apparent depth}}$$

Relation b/w Real height and apparent height

When object is in rare medium and looking from denser medium then object appear away from a normal position.

Applications of Refraction of Light

- i) Sun appear ovale at the time of sunrise and sunset.
- ii) Stars appear twinkle.
- iii) Straight rod deep in water and looking from air then appear bended.

Q-1 What do you mean by lense maker formula? Derive this formula for thin convex lense?

Sol<sup>n</sup> ⇒ Lense maker Formula

That formula which gives relationship b/w focal length, refractive index and radii of curvature of the lense, is called lense making formula.

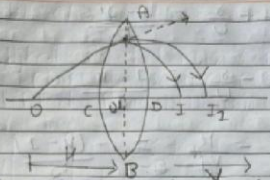
i.e,

$$\frac{1}{f} = (\mu - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$

Where  $f$  = focal length of the lense  
 $\mu$  = Refractive index of lense  
 $R_1$  &  $R_2$  = radii of curvature

Expression for lense making formula:-

Let us consider a thin convex lense having refractive index ' $\mu$ ' and radii of curvature of  $R_1$  and  $R_2$ . Again we suppose 'O' is a point object whose real image is formed at 'I' by the surface 'ACB'. In this case we assume that surface 'ADB' is disappear.



for surface ACB  
 Object distance =  $u$   
 Image distance =  $v_1$

$$\text{Since, } \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{\mu_2}{v_1} - \frac{1}{u} = \frac{\mu_2 - 1}{R_1} \quad \text{--- (i)}$$

But actually surface ADB is appeared final image is formed at I. In this case acts as a virtual object

for surface ADB

$$\mu_1 = \mu$$

$$\mu_2 = 1$$

Image distance =  $v$

Object distance =  $v_1$

$$\text{Since, } \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\Rightarrow \frac{1}{v} - \frac{\mu}{u} = \frac{1 - \mu}{R}$$

$$\Rightarrow \frac{1}{v} - \frac{\mu}{u} = \frac{1 - \mu}{R_2}$$

$$\frac{1}{v} - \frac{\mu}{v_1} = -\frac{(\mu - 1)}{R_2} \quad \text{--- (ii)}$$

equating (i) + (ii)

$$\frac{1}{v} - \frac{1}{u} = \frac{\mu - 1}{R_1} - \frac{(\mu - 1)}{R_2}$$

$$\Rightarrow \frac{1}{f} = (\mu - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$

Wave front  $\Rightarrow$  The locus of the medium particle around thus show which vibrate in the same phase is called wave front.

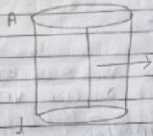
Its shape & size depend upon the nature of source.

Types of wave front

1) Spherical wave front  $\Rightarrow$  When source of light is point then spherical wave front is formed.

(ii) Cylindrical wave front (WF)

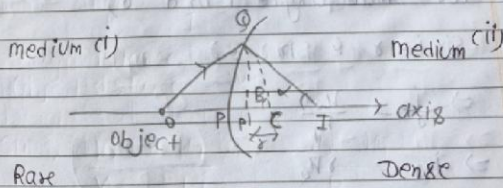
When source of light is as slit then spherical wave front.



(iii) Plane wave front  $\Rightarrow$  When spherical wave covers large distance then it is called plane wave front.

Refraction by spherical surface:

i) For convex surface



Here  $OP \approx OP'$  Object of distance ( $u$ )  
 $PC \approx P'C$  Radius of curvature ( $R$ )  
 $PI \approx P'I$  Image of distance ( $v$ )

In  $\Delta OPT$

$$\alpha + \beta = \theta$$

$$\beta = \theta - \alpha \quad \text{--- (1)}$$

In  $\Delta OQC$

$$\theta + \beta = i$$

$$i = \theta + \beta$$

Here  $\alpha, \beta, \theta, i$  &  $\delta$  are very small angle.

Now, from Snell's Law

$$\frac{\sin i}{\sin \delta} = \frac{\mu_2}{\mu_1}$$

$\therefore$  for small angle  $i$  and  $\delta$

$$\sin i \approx i \quad \& \quad \sin \delta \approx \delta$$

For small angle  $i$  and  $\delta$ ,

$$\sin i = \delta \quad \& \quad \sin \delta = \delta$$

$$\Rightarrow \frac{i}{\delta} = \frac{\mu_2}{\mu_1}$$

$$\Rightarrow i \mu_1 = \delta \mu_2$$

Now putting the value of  $i$  and  $\alpha$  from eq<sup>n</sup> (i) & (ii)

$$(\theta + B) \mu_1 = (B - \alpha) \mu_2$$

$$\Rightarrow \mu_1 \theta + \mu_1 B = \mu_2 B - \mu_2 \alpha$$

$$\Rightarrow \mu_1 \theta + \mu_2 \alpha = (\mu_2 B - \mu_1 B)$$

$$\Rightarrow \mu_1 \theta + \mu_2 \alpha = 0 (\mu_2 - \mu_1) \quad \text{--- (iii)}$$

Now, in  $\Delta OQP'$

$$\tan \theta = \frac{QP'}{OP'} = \frac{QP'}{CP}$$

$\therefore \theta$  is very small angle

$$\therefore \tan \theta \approx \theta$$

$$\theta = \frac{QP'}{CP}$$

Now, in  $\Delta QCP'$

$$\tan B = \frac{QP'}{CP'} = \frac{QP'}{CP}$$

$\therefore B$  is very small angle

$$\tan B \approx B$$

$$\therefore B = \frac{QP'}{CP}$$

Now, in  $\Delta QIP'$

$$\tan \alpha = \frac{QP'}{IP'} = \frac{QP'}{IP}$$

$\therefore \alpha$  is very small angle

$$\therefore \tan \alpha = \alpha$$

$$\alpha = \frac{QP'}{IP}$$

Now putting the value of  $\alpha, B$  in eq<sup>n</sup> (iii)

$$\mu_1 \theta + \mu_2 \alpha = (\mu_2 - \mu_1) B$$

$$\mu_1 \frac{QP'}{CP} + \mu_2 \frac{QP'}{IP} = (\mu_2 - \mu_1) \frac{QP'}{CP}$$

$$\Rightarrow QP' \left[ \frac{\mu_1}{CP} + \frac{\mu_2}{IP} \right] = (\mu_2 - \mu_1) \frac{QP'}{CP}$$

$$\Rightarrow \left[ \frac{\mu_1}{CP} + \frac{\mu_2}{IP} \right] = \frac{(\mu_2 - \mu_1)}{CP}$$

$$\Rightarrow \frac{\mu_1}{CP} + \frac{\mu_2}{IP} = \frac{(\mu_2 - \mu_1)}{CP}$$

$$\Rightarrow \frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{(\mu_2 - \mu_1)}{R}$$

The unit of vergence measurement is  
dioptr  $D = \frac{1}{F}$  (in metre)

Vergence formula

$$V = \frac{1}{q} \text{ (in metre)}$$

$$= \frac{100}{q} \text{ (in cm)}$$

The Focal point is the point at which light rays that pass through a lens or reflected off a curved mirror converge or appear to diverge. It is also known as focus or principle focus.

#### # Equivalent lens:-

If image of an object formed by several lenses at a particular point in certain magnification. If image of that object formed by single lens at the point that magnification then this single lens is called equivalent lens. focal length of this lens is called equivalent focal length.



Let us consider  $O'$  is a point object whose image is formed at  $I$  by the lens  $L_1$  in case we suppose that  $L_2$  lens is disappear.

For lens  $L_1$

$$\text{Object distance} = u$$

$$\text{Image distance} = v_1$$

$$\text{focal length} = f_1$$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\Rightarrow \frac{1}{f_1} = \frac{1}{v_1} - \frac{1}{u} \quad (1)$$

But actually  $L_2$  lens is appear. So final image is formed at  $I$  by the lens  $L_1$  in case we suppose that  $L_2$  lens is disappear.

For lens  $L_2$

Object distance =  $v_2$

Image distance =  $v_1$

Focal length =  $f_2$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\Rightarrow \frac{1}{f_2} = \frac{1}{v_2} - \frac{1}{u_1} \quad \text{--- (i)}$$

But actually  $L_2$  lens is placed so

adding eq<sup>n</sup> (i) & (ii)

$$\frac{1}{f_1} = \frac{1}{v_1} - \frac{1}{u} \quad \text{--- (iii)}$$

If  $F$  be the focal length of equivalent lens

$$\frac{1}{F} = \frac{1}{v} - \frac{1}{u} \quad \text{--- (iv)}$$

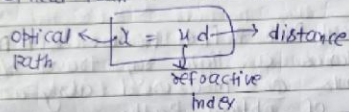
from eq<sup>n</sup> (iii) & (ii)

$$\boxed{\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}} \quad \text{Hence proved}$$

\*) Optical Path  $\Rightarrow$  The distance travelled by light in vacuum in a time  $t$  which it has travelled a distance  $d$  in a given medium is called as optical path.

When a ray of light travel a distance  $d$  in a medium of refractive index  $n$ . The product  $nd$  is called the optical path.

Optical Path  $\propto$  time of travel



Consider there is a medium PQ = Geometrical Path. The time taken by light to cover distance from P to Q =  $t$  whose refractive index is  $n$ .

In vacuum  $P' \rightarrow Q'$

$\rightarrow t \leftarrow$

$P'Q' = \text{Optical Path}$

The speed of light in a medium =  $\frac{\text{distance travelled}}{\text{time}}$

$$v_c = c = \frac{d}{t}$$

$$v_m = \frac{d}{t}, \quad v_c = \frac{d}{t}$$



$$t = \frac{d}{v_m}, \quad t = \frac{x}{c}$$

$$\frac{x}{c} = \frac{d}{v_m} \quad \left\{ \text{but } v = \frac{c}{\mu} \right\}$$

$$x = \frac{cd}{v_m} = \frac{c}{v_m} d$$

$$\Rightarrow x = \mu d$$

# Geometrical Path  $\Rightarrow$  The geometrical path is the distance a light ray travel b/w two points as measured along a segment of optical path.

Sign convention

Left  $\longrightarrow$  Right

- Incident light will be considered as travelling from left to right.
- An arrow ahead a ray indicate the direction in which light is travelling.
- All distance (object distance, image distance, focus length) are the major from optical system.

- Distance major from left of optical axis is consider as negative.
- Distance major above the principal axis is consider as positive.
- Distance major below the principal axis is consider as negative.
- Angle of incident, refraction and reflection of measure from the normal to the ray.
- Angle measured as anticlockwise direction or considered as positive and measure as clockwise direction or negative.

# Lateral magnification:- (Spherical surface)

Lateral magnification or linear magnification or transverse magnification ( $m$ ) is defined as the ratio of size of image to the size of object.

$$m = \frac{\text{Size of image}}{\text{Size of object}}$$

Let a spherical reflecting surface MPV separating two medium refractive index  $n_1$  and  $n_2$ . Form  $\triangle ABC$  &  $A'B'C$ .

$$\frac{A'B'}{AB} = \frac{A'C}{AC} \quad \text{--- (i) } A'B' = -v_2$$

$$-v_2 = \frac{v-r}{-u+r} \quad \begin{aligned} AB &= y_1 \\ A'C &= PA' - PC \\ &= v-r \end{aligned}$$

$$AC = AP + PC = -u + r$$

Multiplying with negative

$$\frac{v_1}{u} - \frac{v_2}{v} = \frac{n_1 - n_2}{r}$$

$$\frac{n_1}{u} - \frac{n_2}{v} = \frac{n_1}{r} - \frac{n_2}{r}$$

$$\frac{n_1}{u} - \frac{n_2}{v} = \frac{n_1}{r} - \frac{n_2}{r}$$

$$\frac{n_1}{u} - \frac{n_2}{r} = \frac{n_2}{v} + \frac{n_2}{r}$$

$$n_1 \left( \frac{r-u}{ur} \right) = n_2 \left( \frac{r-v}{v r} \right)$$

$$\Rightarrow \frac{n_2}{n_1} \times \frac{vr}{ur} = \frac{r-u}{r-v}$$

$$\Rightarrow \frac{n_2}{n_1} \times \frac{v}{u} = \frac{r-u}{r-v} \quad \text{--- (iii)}$$

From eqn (iii) & (iv)

$$\Rightarrow \frac{n_1}{n_2} \times \frac{v}{u} = \frac{y_2}{y_1}$$

$$\Rightarrow \frac{y_2}{y_1} = \frac{n_1}{n_2} \times \frac{v}{u}$$

$$\therefore M = \frac{n_1}{n_2} \left( \frac{v}{u} \right)$$

# Axial magnification (longitudinal) (Spherical surface)

In general objects are not linear but distributed or extended therefore in addition to lateral magnification or longitudinal magnification also occurs.

$$L = \frac{dv}{du} = \frac{\text{length of image along axis}}{\text{length of object along axis}}$$

It is defined as the ratio of length of image along axis to length of object along axis.

from refraction formula

$$\frac{n_1}{u} - \frac{n_2}{v} = \frac{n_2 - n_1}{R}$$

differentiation both sides

$$n_1 \left( \frac{-1}{v^2} \right) - n_2 \left( \frac{-1}{u^2} \right) \left( \frac{dv}{du} \right) = 0$$

$$\Rightarrow \frac{n_1}{v} + \frac{n_2}{u} \left( \frac{dv}{du} \right) = 0$$

$$\Rightarrow \frac{n_2}{u} \left( \frac{dv}{du} \right) = \frac{n_1}{v}$$

$$\Rightarrow \frac{dv}{du} = \frac{n_1^2}{n_2^2} \cdot \frac{v^2}{u^2} \times \frac{n_2}{n_1}$$

$$\Rightarrow \frac{dv}{du} = \frac{n_1^2}{n_2^2} \cdot \frac{v}{u}$$

$$L = \frac{n_1^2}{n_2^2} \cdot \frac{v}{u}$$

# Cardinal points or planes: → In 1841, # Gauss

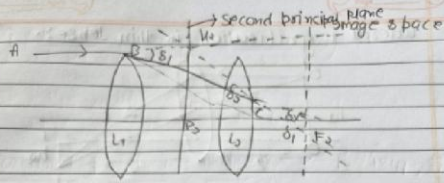
Show that any number of ~~separate~~ co-axial lenses could be treated as a single unit without the necessity of treating the single surface of lenses separately. The lens formula can be applied to the system provided the distances are measured from two hypothetical parallel planes. These planes are known as cardinal planes and the point of intersection of these planes with the axis are called the principal point or Gauss point.

There are six points which are characterized optical system.

- (i) Two focal point
- (ii) Two principle point
- (iii) Two nodal point

1) Principle points and principal planes: →

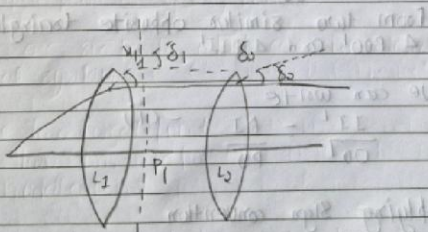
Principle point of an optical system is defined as the point from where assume refraction to occur without refraction as to where the



$H_2P_2$  is second principal plane and  $F_2$  is second principal point.

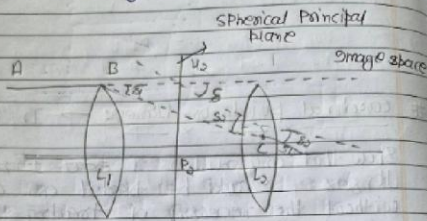
\* First principal point or First principal focus:

A beam of light passing the point  $F_1$  on the axis of the object side and after refraction to the principal axis then the point  $F_2$  is called second principle focus.



Plane  $H_1P_1$  is called First principal plane and point  $P_1$  is First principal point.

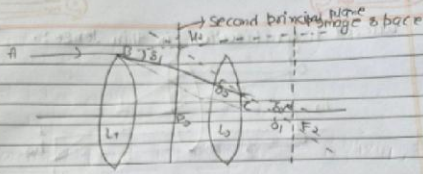
refraction actually occurs.



$H_2P_2$  is second principal plane and  $P_2$  is second Principal point.

iii) Focal Point and Focal Planes: →

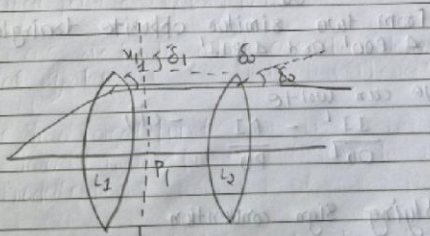
If a parallel beam of light traveling from left to right is incident on optical system then the beam comes together at a point  $F_2$  on the other side of the optical system (Image space) the point  $F_2$  on the other side of the  $F_2$  is called the second point and the plane passing through it is called second focal plane.



$H_2$  is second principal plane and  $F_2$  is second principal focus.

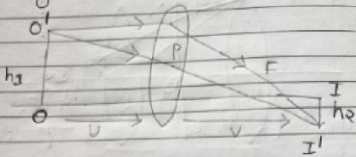
\* First principle point or First principal focus:-

A beam of light passing the point  $F_1$  on the axis of the object side and after refraction to the principal axis then the point  $F_2$  is called second principle focus.



Plane  $H_1$ ,  $P_1$  is called first principal plane and point  $F_1$  is first principal point.

# Lateral magnification in thin lense:-



Where  $h_1$  is height of object at  $OO'$   
 $h_2$  is the height of inverted real image at  $II'$  and  $OP$  in the ray of passing through pole  $P$ .

The lateral or transverse magnification is defined as

$$M = \frac{II'}{OO'} \quad \text{--- (i)}$$

From two similar opposite triangle  $\triangle POO'$  and  $\triangle PII'$

We can write

$$\frac{II'}{OO'} = \frac{PI}{PO} \quad \text{--- (ii)}$$

Applying sign convention

$$\frac{-h_2}{h_1} = \frac{v}{-u}$$

Therefore equation is becomes

$$m = \frac{h_2}{h_1} = \frac{v}{-v}$$

$$m = \frac{h_2}{h_1} = \frac{v}{v}$$